*Salud, Ciencia y Tecnología*. 2025; 5:2643
doi: 10.56294/saludcyt20252643

AG EDITOR

**ORIGINAL**

# The Impact of Visualization Integration and Mason's Theory in Real Analysis Learning on Students' Mathematical Proof Abilitie

# El impacto de la integración de la visualización y la teoría de Mason en el aprendizaje del análisis real sobre las habilidades de demostración matemática de los estudiantes

Dewi Murni[1], Yerizon[1], Elita Zusti Jamaan[1], I Made Arnawa[2], Helma[1], Mirna[1]

[1]Universitas Negeri Padang, Mathematics. Padang, Indonesia.
[2]Universitas Andalas, Mathematics. City, Country.

**Cite as**: Murni D, Yerizon, Zusti Jamaan E, Arnawa IM, Helma, Mirna. The Impact of Visualization Integra-tion and Mason's Theory in Real Analysis Learning on Students' Mathematical Proof Abilitie. Salud, Ciencia y Tecnología. 2025; 5:2643. https://doi.org/10.56294/saludcyt20252643

**Submitted:** 22-08-2025 **Revised:** 03-10-2025 **Accepted:** 23-12-2025 **Published:** 24-12-2025

**Editor:** Prof. Dr. William Castillo-González

## ABSTRACT

**Introduction:** mathematical proof abilities are crucial for mathematics students because mathematics is full of theories and concepts that must be proven. Mathematical proof is the core of the Real Analysis course. However, most students struggle with this course because the topic consists of abstract concepts and proofs.
**Objective:** providing visualizations of abstract concepts and Mason's framework of thought is expected to be a solution to help students conduct proofs.
**Method:** this research is a descriptive qualitative study. The research instruments were a mathematical proof test in the form of an essay and a structured interview guide. The research subjects were: one class of 35 students of the Mathematics Education study program, Faculty of Mathematics and Natural Sciences, Padang State University (Indonesia). For the interview, 3 people were taken from the lower group and 3 people from the upper group. Data analysis was carried out using descriptive analysis.
**Results:** the results of the mathematical proof test showed that there was an increase in students' mathematical proofs before and after learning that integrated visualization and Mason's framework of thinking. From the interview results, it was found that: for students in the lower group, the use of visualization could improve their understanding of abstract concepts but they were not yet able to write proofs well. Meanwhile, students in the upper group, the use of visualization could improve their understanding of abstract concepts and write proofs and describe them in the good and very good categories.
**Conclusions:** use Mason's visualization and conceptual framework in learning can improve students' mathematical proofs. Although the improvement is not significant, lower-level students saw improvements at the cognitive level of understanding, while higher-level students saw improvements at the application and analysis levels.

**Keywords:** Visualization; Mason's Theory; Mathematical Proof; Real Analysis.

## RESUMEN

**Introducción:** las habilidades de demostración matemática son cruciales para los estudiantes de matemáticas, ya que esta disciplina está repleta de teorías y conceptos que deben demostrarse. La demostración matemática es fundamental en el curso de Análisis Real. Sin embargo, la mayoría de los estudiantes tienen dificultades con este curso debido a que el tema se compone de conceptos y demostraciones abstractas. Se espera que proporcionar visualizaciones de conceptos abstractos y el marco de pensamiento de Mason sea una solución que ayude a los estudiantes a realizar demostraciones.
**Objetivo:** analizar el impacto del uso de la visualización y el marco de pensamiento de Mason en las habilidades

de demostración matemática de los estudiantes.
**Método:** esta investigación es un estudio cualitativo descriptivo. Los instrumentos de investigación fueron una prueba de demostración matemática en forma de ensayo y una guía de entrevista estructurada. Los sujetos de investigación fueron: una clase de 35 estudiantes del programa de estudios de Educación Matemática de la Facultad de Matemáticas y Ciencias Naturales de la Universidad Estatal de Padang (Indonesia). Para la entrevista, se seleccionaron 3 estudiantes del grupo de menor nivel y 3 del grupo de mayor nivel. El análisis de datos se realizó mediante análisis descriptivo.
**Resultados:** los resultados de la prueba de demostración matemática mostraron un incremento en las demostraciones matemáticas de los estudiantes antes y después del aprendizaje que integró la visualización y el marco conceptual de Mason. Las entrevistas revelaron que, para los estudiantes del grupo de menor rendimiento, el uso de la visualización mejoró su comprensión de conceptos abstractos, pero aún no eran capaces de redactar demostraciones con claridad. En cambio, para los estudiantes del grupo de mayor rendimiento, el uso de la visualización mejoró su comprensión de conceptos abstractos y les permitió redactar demostraciones y describirlas como buenas o muy buenas.
**Conclusiones:** el uso de la visualización y el marco conceptual de Mason en el aprendizaje puede mejorar las demostraciones matemáticas de los estudiantes. Si bien la mejora no fue estadísticamente significativa, los estudiantes de menor rendimiento mostraron avances en el nivel cognitivo de comprensión, mientras que los de mayor rendimiento mejoraron en los niveles de aplicación y análisis.

**Palabras clave:** Visualización; Teoría de Mason; Demostración Matemática; Análisis Real.

## INTRODUCTION

Mathematical proof is a process that hones students' analytical thinking logically and systematically based on sound concepts. This process is essential in today's era of progress. Proof is not just about following logical steps, but also about understanding and applying fundamental concepts of mathematics[1] with precision and accuracy.[2] Proof is a logical explanation of why the statement is true.[3,4] Furthermore [5,6] stated that the best proof is not just saying that a statement is true but explaining why the statement is true. Proof is the basis of mathematics[7] Mathematical proof is a key element in advanced mathematics education and is the main focus of Real Analysis courses. However, most students experience difficulties in this course. One previous research[8] found that students have difficulty in solving problems because the topic is abstract and full of symbols.

Example definition suppose $X = (x_n)$ a sequence of real numbers. A real number x it is said the limit of $(x_n)$, if for each number $\varepsilon>0$. There is a natural number $K(\varepsilon)$, for all $n \geq K(\varepsilon)$, terms $x_n$ satisfied l $|x_n-x|<\varepsilon$. Equivalent: $\lim(x_n)=x \leftrightarrow \forall\varepsilon>0\ \exists K(\varepsilon)\epsilon N \ni \forall n\geq K \rightarrow |x_n-x|<\varepsilon$.

In the compulsory student reference book, explanations are given only in analytical form, making them difficult for students to understand. Clark et al.[9] stated that to help students' understanding, sometimes sentences and oral text alone are not enough, but it would be better if visual graphics were added, and providing graphics that are relevant to the text is a method that has been proven to encourage stronger cognitive processes in students. Furthermore, research results[10] revealed that there was around 20 % difference in the percentage of students who answered correctly in tests if the learning used words and graphics compared to learning using words alone. Several studies in Indonesia[11,12] also found that the use of visual media has been proven to improve the quality of learning. Therefore, providing visualizations in learning is an alternative to addressing student difficulties. Visualizations allow students to see the meaning of concepts and symbols more clearly, making them easier to grasp.

Another difficulty experienced by students is the difficulty in carrying out the proof process (mathematical proof) for both theorems and problems. Students sometimes find it difficult to start a proof and the proof process is unstructured.[13] Furthermore, [6] explains four main difficulties faced by students in proof, namely: i) difficulty understanding mathematical concepts, ii) difficulty in mathematical language and notation, iii) lack of mathematical proof strategies, and iv) difficulty in reading mathematical proofs. Furthermore, [14] states that undergraduate mathematics students often face various difficulties in understanding and constructing mathematical proofs. To overcome these difficulties, students need to understand each related definition and theorem and be able to organize these concepts. The use of appropriate learning strategies or approaches is also important to help students in carrying out the proof process. Mason's framework is one approach that students can use to help students systematically carry out their proof process. Mason 's framework consists of three stages, namely: Entry, Attack and Review/recheck.[15,16] In the entry stage, students identify problems by carefully reading the question, determining relevant ideas, focusing on their own findings, classifying brief information, and representing it in the form of notation or symbols. The attack stage is marked by the emergence of conjectures or justifications that indicate confidence in a problem. Claim. The recheck stage is characterized by students actively checking the accuracy of the solution. The solution matches the question, provides reasons to ensure the correct solution, and provides the implications of the conjecture or argument.

Based on the problems, to improve students' mathematical proof abilities in Real Analysis lectures (especially on the topic of Real Number Sequences) is to integrate two activities, namely providing visualization and Mason's Thinking Framework in learning. Providing visualization will be able to clarify abstract concepts and symbols used while the Mason-thinking framework helps students think systematically and logically. Therefore, the purpose of this study is to analyze the impact of providing visualization and using the Mason Thinking Framework on students' mathematical proof skills.

## METHOD

This research is a qualitative study. The study was conducted on a class of 35 students in the Mathematics Education study program, Faculty of Mathematics and Natural Sciences, Padang State University, located in Padang, West Sumatra Province, Indonesia. The study was conducted from January to June 2025. The sample was taken using total sampling, while interviews were selected in stages, with 3 students from the lower group and 3 students from the upper group. The observed and analyzed variables were mathematical proof ability, as seen in the proof construction indicator. Proof construction is reasoning from proven facts (premises) using appropriate properties (e.g., definitions, axioms, proven theorems) and logically valid steps to arrive at a conclusion.[17] The research instruments were: a mathematical proof ability test in the form of essay questions and an interview guide sheet.

The data from the proof-making ability test were analyzed using: (i) a score ranging from 0 to 100 and (ii) a qualitative assessment using the proof construction indicator with 7 sub-indicators following the Entry-Attack-Recheck steps. The results were then assessed in the following categories: excellent, good, less excellent, and poor. The categories were: excellent if the student wrote all answers correctly; good if the student wrote most of the answers correctly; less excellent if the student wrote only a small number of answers correctly; and poor if the student wrote no answers or all of the answers were incorrect. The interview guide consisted of five questions related to the students' learning activities. Because this research is part of the researcher's doctoral dissertation, all instruments and procedures have been validated by lecturers who are experts in their fields. The interview data will be analyzed using descriptive analysis, which means the data is presented or narrated as is.[18]

The learning process generally consists of four stages:

Stage 1: learning begins with understanding the material to be studied. Students are provided with reading materials organized according to the syllabus and required reference books. Visualizations are then added to the books to further explain the meaning of the concepts. For example, consider the concept of the limit of a sequence.

$\lim(x_n)=x \leftrightarrow \forall\varepsilon>0\ \exists K(\varepsilon)\epsilon N\ \exists\forall n\geq K \rightarrow |x_n-x|<\varepsilon$

So the visualization can be seen as follows:

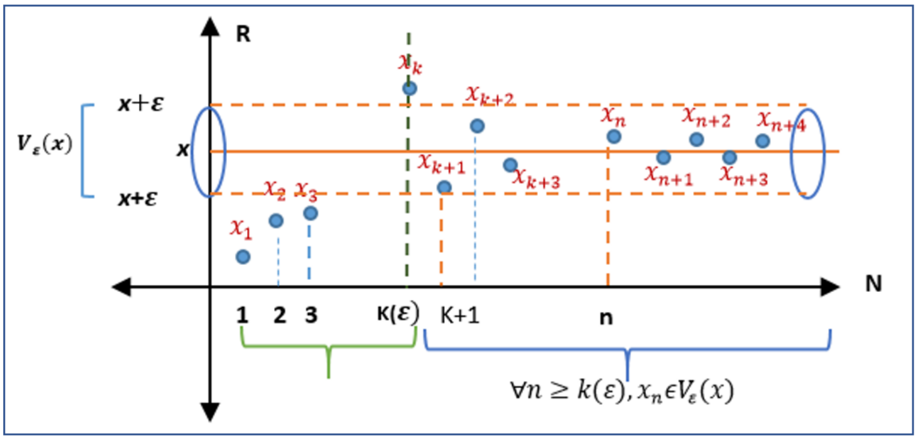



**Figure 1.** Visualization for $\lim(x_n)=x$

This definition can be depicted on two coordinate axes. The horizontal axis shows the nth term and the vertical axis shows the value of the sequence $x_n$ (figure 1).

The position of the blue dots shows the values of the sequence $(x_n)$ which increases and decreases from the 1st term, the 2nd term and so on to the right, $x_n$ approaching the x line. This image shows that the limit of the sequence ($x_n$) is x.

From figure 1, it can be seen that for a ε>0, there is a limit to the Kth term (ε) which depends on the value ε so that all sequences $x_n$ to the right of the Kth term are all in the interval (x-ε, x+ε) = $V_ε$ (x) or all purple points lie between the line x-ε and the line x+ε. Because ε>0 is a number that can be very small, the value of the sequence ($x_n$) approaches x. Next, during the lesson, the lecturer displays visualizations at the front of the class. Using a question-and-answer method (class discussion), students understand the material.

In stage 2, the lecturer provides exercises using Mason's framework (Entry-Attack-Recheck) and provides questions to guide students' thinking in conducting the proof. The exercises are conducted in discussion groups.

Example: prove the limit of the following sequence using Mason's framework.

$\lim(n/(n^2+1))=0$

Some questions asked to guide the answers are:

- Entry: (To understand the question, answer the following question first))
  1. Q: What things are known from the question?
  2. Q: What things are asked in the question?

- Attack: (Answer the following questions to guide you in completing the proof)
  1. Q: What is the meaning/definition $\lim(n/(n^2+1))=0$ in notation? **ε-k** (which will be proven)?
  2. Q: What is done in the preliminary analysis?, what will be looked for?
  3. Q: Write a formal proof according to the definition of the limit of a sequence.
  4. Q: Write a conclusion.

- Recheck: (recheck the proof steps)
  1. Q: Does the answer match the theory? Or is the theory used appropriate?
  2. Q: Are the proof steps systematic?
  3. Q: Has the question been answered?

However, during the exam, students were no longer given guiding questions.

Stage 3: students present their group work to the class. The presentations must be reviewed by other groups to determine whether the answers are correct. At the end of the lesson, the instructor provides reinforcement.

## RESULTS

To determine the impact of learning that integrates visualization and the Mason Thinking framework on mathematical proof abilities, at the end of the learning process, students were given a test with two proof problems. From the test results with a range of 0-100, the average student learning outcome was 56,8 with a standard deviation of 21,2, which was higher than the average previous learning outcome with an average of 35,5 with a standard deviation of 14,9 for learning without visualization and the Mason framework. As shown in figure 2 below.

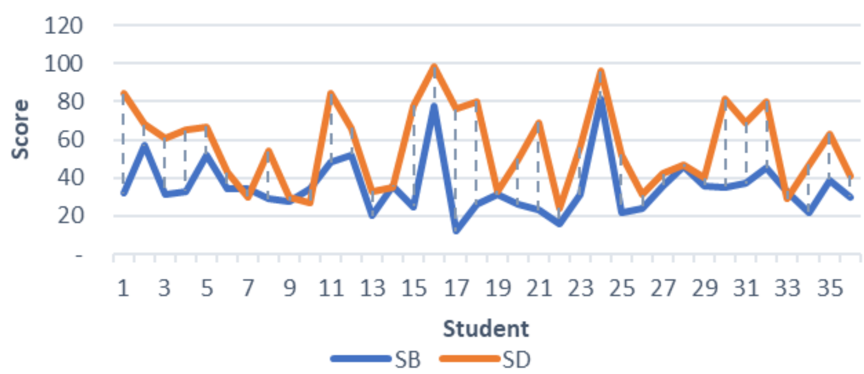



**Figure 2.** Test results before (SB) and after (SD) the study

Figure 2 shows that before the study, students' mathematical proof skills and learning outcomes were very low. With this study, there appears to be an improvement in learning outcomes. Overall, there has been an increase in student grades, although some students have not seen significant changes.

Furthermore, to examine the proof process, students' answers to one question with 7 indicators were examined.

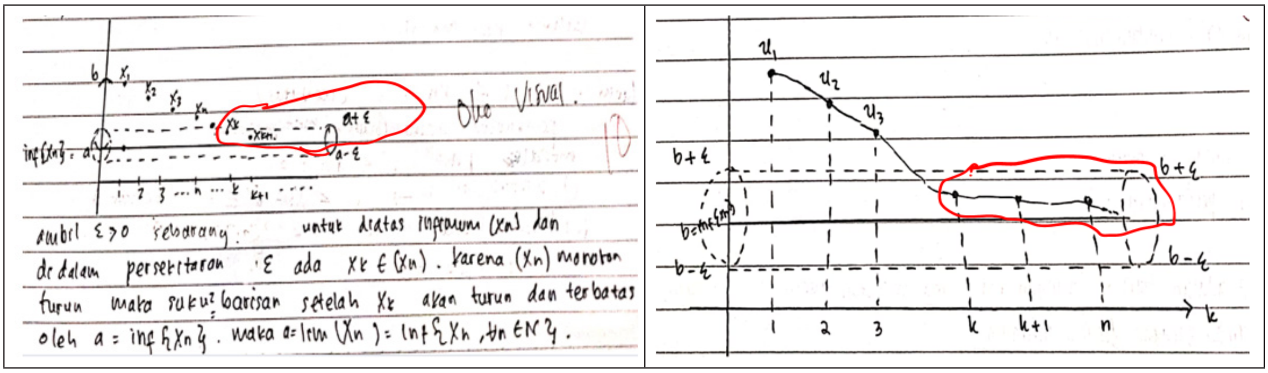
Prove: a. If X = ($x_n$) is a monotonically decreasing and finite sequence of real numbers then lim($x_n$)=inf{$x_n$}. b. Provide a visual illustration of the problem.

The test results were analyzed based on seven indicators of "proof construction" and divided into four categories: excellent, good, poor, and poor. The results of the Entry stage proof test can be seen in table 1.

**Table 1.** Percentage of students answering proof questions at the Entry stage with the following categories: Very Good, Good, Less Good and Bad

| Research Indicators Symbol | Very good | Good | Not good | Bad |
|---|---|---|---|---|
| Writing the premises correctly (E1) | 76,5 | 8,8 | 2,9 | 11,8 |
| Write down what will be proven (E2) | 73,5 | 17,6 | 2,9 | 5,9 |
| Can sketch graphs (E3) | 26,5 | 23,5 | 32,4 | 17,6 |
| The initial steps of proof include using the correct proof method (A1) | 32,4 | 38,2 | 14,7 | 14,7 |
| Using a systematic flow of evidence (A2) | 26,5 | 38,2 | 23,5 | 11,8 |
| The concept/property used is correct (A3) | 29,4 | 32,4 | 23,5 | 14,7 |
| Giving correct reasons in the proof steps (A4) | 29,4 | 26,5 | 26,5 | 17,6 |
| Making conclusions (A5) | 26,5 | 26,6 | 20,6 | 23,5 |
| Checking the suitability of the theory (R1) | 32,4 | 26,5 | 8,8 | 32,4 |
| Checking the suitability of the answer to the question (R2) | 35,3 | 26,5 | 8,8 | 29,4 |

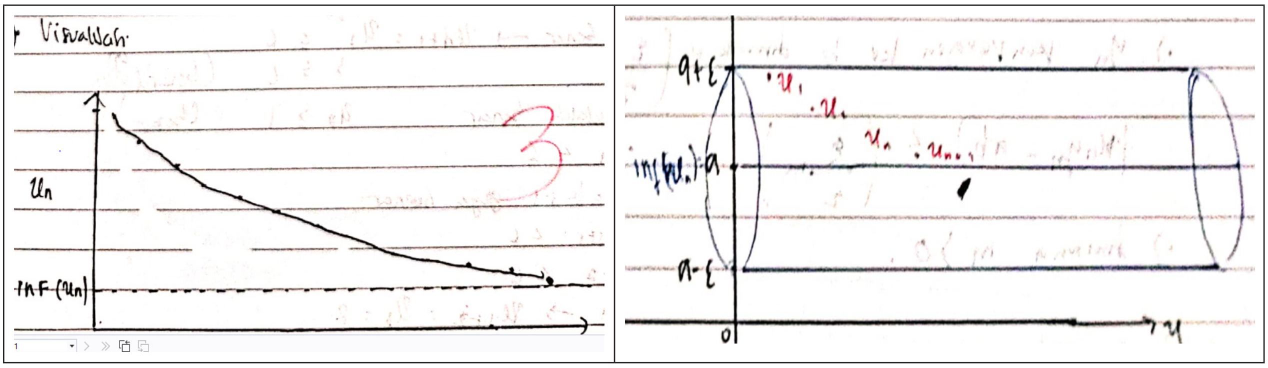
From table 1, it can be seen that 85,3 % of students' answers to indicator E1 (writing premises correctly) fall into the categories: very good and good, This data shows that most students have a good understanding of what is known as a premise, but there are still 15 % (5 people) who write it incorrectly, In indicator E2 (Writing the thing to be proven in symbols) Almost all of them were good, namely 91,2 %, the remaining 8,8 % (3 people) were wrong, Thus, almost all of the students had written the correct thing that will be proven and writing symbols correctly, From indicators E1 and E2, it can be concluded that students have been able to understand the questions well, on the E3 indicator (sketching the graph) 26,5 % of students were categorized as very good and 23,5 % as good. The graph drawing indicator shows that half of the students were able to draw graphs in the good category (figures 3a and 3b).




(a) (b)

**Figure 3.** Sketch of the graph of lim($x_n$)=inf{$x_n$} the upper group of students

In figure 3a, it can be seen that students are able to draw the terms of the descending sequence but do not exceed the infimum line as the largest lower limit (because it is limited). They have created a neighborhood of the infimum value a and for all n>k, $x_n \in$ (a-ε, a+ε). This is in accordance with the definition of the limit of the sequence. However, the infimum function has not been explained even though it has been well described. In figure 3b, it can also be seen that students have been able to create a good visualization similar to figure 3a. The difference is that in figure 3a, the $x_k$ initial term enters the neighborhood while in figure 3b, $x_{k+1}$ the initial term enters the sequence. Both are correct because they are in accordance with the definition of the limit of the sequence. Meanwhile, the visualization sketch of the problem from the lower group students can be seen in figure 4 below.



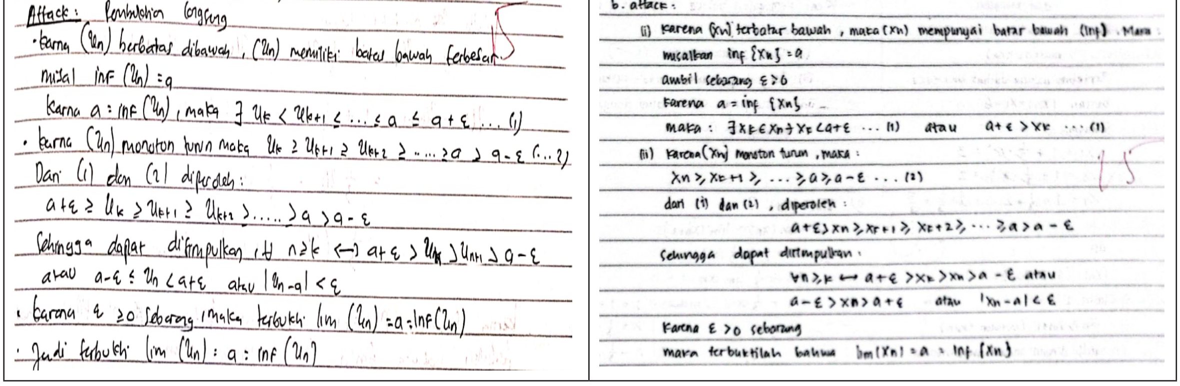
(a) (b)

**Figure 4.** Sketch of the graph $\lim(x_n)=\inf\{x_n\}$ of students in the Lower group

From figure 4a, it can be seen that the students in the lower group are still at the stage of depicting ($x_n$) descending and limited sequences but have not been able to depict according to the definition of the sequence limit. Meanwhile, in figure 4b, students understand the wrong concept where the epsilon should represent a very small environment. So that later all the terms of the sequence approach the limit value of the sequence. The infimum function is not yet visible in the image so that there is no limit K(ε). From figure 4a and figure 4b, it can be seen that students' understanding of the environment and infimum is still weak so that they are also weak in depicting it. Meanwhile, based on observations, the visualization shown by the lecturer in front of the class has been able to make it easier for students to understand the topic but they still have difficulty in drawing it.

Next, it can be seen on the indicators (A1 – A5). On A1 there are 70,6 % of students are in the very good and good category, the rest 29,4 are less good and bad. This means that most students have started the proof correctly, that is, they have started with the correct concept. Only a little number of students who start the proof incorrectly. Furthermore, in A2 (using a systematic proof flow) 64,7 % are in the very good and good category. This data mean that most students have been able to start with the correct concept and sequence the proof flow well. The remaining one-third of students (35,3 %) are in the less good and bad category who cannot make the proof flow correctly where the proof flow carried out still jumps and the use of the concept in reverse order. In A3 (The concept/property used is correct) 61,8 % are in the very good and good category; and the remaining 38,2 are in the less good and bad category. This fact shows that if students have started correctly, the concept they use for proof is also correct. Although there are still some students who have difficulty understanding the concept, especially in understanding the infimum. They often use inequality signs incorrectly. In A4 (Giving correct reasons in the proof step) 55,9 % are in the very good and good category. This data mean that about half of the students have been able to make correct reasons from the proof they made. Some are still less good and bad, where with the wrong concept, so the proof is also wrong. In A5 (Making conclusions) 52,9 % are in the very good and good category. This finding means that more than half of the students have been quite successful in making proofs to conclusions correctly. While other students' conclusions made are not based on logical conclusions. So in general, at the Attack stage, most students (above 50 %) are in the very good and good category in conducting proofs on each indicator, especially in terms of starting the proof. The rest are still weak in proof.

The following are the answers of two student in the Good (figure 5a) and Very Good (figure 5b) categories at the Attack stage.



(5a) (5b)

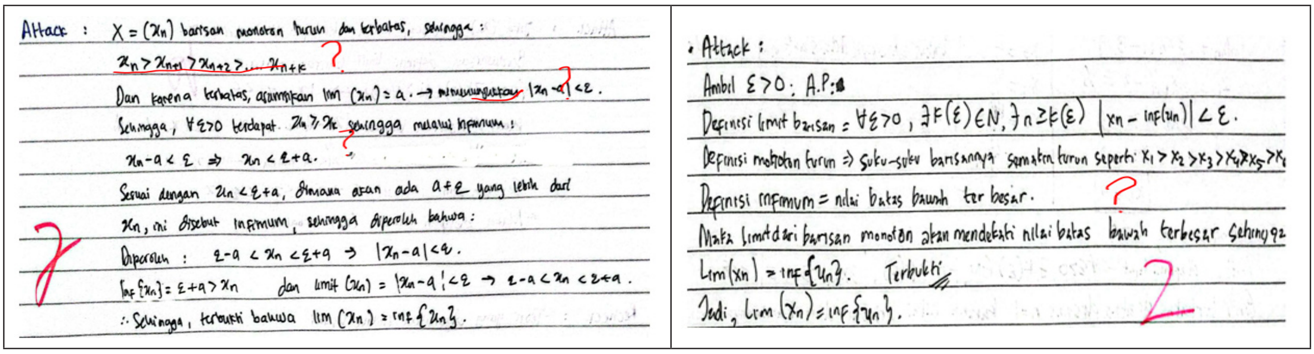
**Figure 5.** Student group answers to the Attack stage

https://doi.org/10.56294/saludcyt20252643 ISSN: 2796-9711

From figure 5a, it can be seen that the students have been able to start the proof well, namely starting from the definition of a finite sequence, which means it is also limited below so that it has an infimum. Next, they make an example of the infimum of the sequence with a . However, there is still a slight error in the definition of the infimum because it is mixed with the definition of a descending sequence. It should be because inf $(x_n)$= a then for ε>0 there will be $x_k$<a+ε. The proof process is systematic until finally finding ∀n≥k maka $|x_n$-a|<ε. Because ε>0 it is arbitrary, the conclusion is correct, namely lim$(x_n)$=a=inf$\{x_n\}$. Furthermore, in figure 5b, the student's answer is in the very good category. The concepts used are correct, namely the infimum concept and the descending sequence concept.

The concept is well explained with the correct symbols. The work is systematic, with equations 1 and 2 arranged sequentially. The reasoning for the conclusion is correct and thus proven. lim$(x_n)$=a=inf$\{x_n\}$.

The following are the errors made by students in the lower group at the Attack stage (figures 6a, 6b).




(6a) (6b)

**Figure 6.** Bottom group's answers at the Attack stage

It can be seen in figure 6a that the mistake made by students at the beginning of the proof was that the concept used was not clear whether the sequence was descending or the sequence was limited to $x_n$<$x_{n+1}$<. $x_{n+2}$. Next lim$(x_n)$=a it is used as a premise even though it is something that will be proven. Then the concept of the environment of a is also wrong a-ε<xn<a+ε. and finally the conclusion drawn is also wrong. Figure 6b shows that the students' proofs do not use premises correctly. They do not use the concepts of finite sequences or infimums. They only translate the concept of descending sequences $x_n$>$x_{n+1}$ but are unable to connect it to the properties of infimums. Therefore, the proofs are far from correct. Therefore, it appears that the proofs performed by students in the lower group are not using concepts correctly and are not systematic.

Furthermore, for indicator R1 (Checking the suitability of the theory), there was 58,8 % and for R2 (Checking the suitability of answers to questions), there was 61,8 % in the very good and good categories. Previously, students rarely did the activity of double-checking. However, in this study, most students have begun to get used to double-checking the results of their work or the proof process they have carried out.

### Interview

Interviews were conducted in order to obtain direct information from students regarding the answers they gave and the problems they experienced in conducting the proof. Interviews were conducted on 6 students, namely: 3 people from the upper group (KA) and 3 people from the lower group (KB). Interview indicators: 1) Understanding the Problem/Question, 2) Causes of Errors, 3) Application of entry-attack-recheck, 4) Effectiveness of visualization in learning, 5) Student learning methods.

The following are the results of the interviews conducted: related to the issues that were proven.

P1: from a proof problem, can you determine:

- What is the premise/hypothesis/known question and
- What will be proven?

KA: the three students can determine the premises and what is known.

- A premise is some statement that is known before the question. For example, given: (i) $(x_n)$ is a descending sequence. (ii) $(x_n)$finite sequence
- The thing that will be proven is the same as what is being sought as an answer. For example Prove lim$(x_n)$=inf$\{x_n\}$

KB: there was one student who wrote the premise incompletely. but all three can write down what they know.

- So the three students were able to determine the premises and what was known was that they were not careful enough in placing their positions.

P: what obstacles did you experience in completing the proof problem?
KA: Obstacles felt:

- Sometimes I'm not sure where to start proving something.
- The symbols used are sometimes reversed/inappropriate.
- Making visualizations is a bit difficult

KB: Obstacles faced:

- Confused about starting the proof
- The concept is poorly understood for reasons of proof, for example the infimum concept cannot be applied to the discussion of the problem.
- Unable to connect between steps in the proof
- The visualization explained by the lecturer can be understood but describes the difficulties themselves.

P3: the Entry-Attack-Recheck (EAR) verification and question guide help you with the verification process?
KA: the proof process and guiding questions are very helpful in terms of:

- The EAR process helps to think systematically step by step.
- Guiding questions serve as a reference for conducting proof, for example, "what is the preliminary analysis?"

KB: the EAR verification process and question guide are very helpful in terms of:

- The EAR process helps us to think and work according to the existing stages.
- Guiding questions serve as a guide for conducting the evidence, for example, "What is the preliminary analysis?" So, by answering that question, we already know what to do.

However, it is still difficult to understand and carry out the proof because of the lack of practice in working on proof problems.

P4: does providing visualization help your understanding of the concept?

KA: it really helps us to understand abstract concepts and try to re-describe what has been understood.

KB: helps us to understand the concept but to describe it again is still difficult.

P5: how do you study?

KA: rewrite the topic that has been studied at home. Group study to complete assignments given by the lecturer.

KB: rarely repeat lessons, prioritizing lessons received on campus only.

Often study alone and rarely study in groups.

If you don't understand, you will be reluctant to ask your seniors so you will only expect an explanation from the lecturer.

From the interview results, it can be concluded that the use of visualization in learning can improve students' understanding of concepts or material, and that the use of Masonic thinking stages can facilitate the proof process. The difficulties experienced by students in the proof process are primarily due to their learning styles being suboptimal. Students lack practice working on structured tasks or independent assignments, making the material difficult to grasp. Furthermore, the lack of utilization of group work makes the difficulties experienced difficult to resolve.

## DISCUSSION

This study was conducted with the aim of addressing the problem of students' low mathematical proof abilities in Real Analysis courses. Some causes include very abstract concepts or materials and symbols. The results of the study indicate that the use of visualization in learning has been able to improve students' understanding of abstract and symbolic concepts. Students have been able to understand the meaning of symbols with the help of visualization. This is very important because the proof has a structure including justification for each step[19] as well as the use of appropriate symbols and mathematical expressions.[17] In addition, it also follows deductive steps and writes these steps systematically.[20] The results of this study indicate that more than half of the students have been able to describe concepts with very good and good categories. This is in line with research [21] which states that visualization is proven to be able to bridge students' understanding of symbolic representations so that students' understanding of the material becomes better. Visualization can be a learning medium that can facilitate students' understanding of the material.[22] The impact of this visualization is also shown by the increase in learning outcomes for most students during learning that integrates visualization and the Mason framework compared to before the study. Similarly, the use of GeoGebra visualizations has also been shown to improve students' mathematical understanding.[23]

The use of the Mason framework and guiding questions has trained students to create proofs systematically. The Mason framework provides students with direction regarding the steps required in the proof process. The guiding questions, meanwhile, serve as a guiding principle for students to determine the concepts to be used in the proof steps. Generally, understanding the problem (entry) involves understanding the premises and conclusions to be proven, as well as their symbols. This part is crucial to ensure students do not misguidedly conduct the proof. Next, in the attack stage, they carry out the proof process by explaining the facts. The premises are related to the concepts and logic of the proof, as well as the conclusion drawn. At the end of the proof process, students are trained to habitually double-check their work. Overall, the research results indicate that some students are still imperfect in the proof process, but there has been improvement. Furthermore, challenges in understanding the concept were identified, including students' low ability to create graphs, as found by Yerizon[24].

This research was hampered by time constraints and students' lack of practice outside of class hours. Future research is recommended to optimize the time and assignments students must complete.

## CONCLUSIONS

Based on the research data, it can be seen that the use of visualization in Real Analysis learning can improve students' understanding of concepts and symbols. Meanwhile, Mason's framework of thought can improve the proof process carried out by students systematically and logically. Therefore, it can be concluded that providing visualization and Mason's framework of thought can improve students' mathematical proof in Real Analysis courses.

## ACKNOWLEDGEMENTS

The authors would like to thank Lembaga Penelitian dan Pengabdian Masyarakat Universitas Negeri Padang for funding this work with a contract number: 1675/UN35.15/LT/2024 fiscal year 2024.

## BIBLIOGRAPHIC REFERENCES

1. Radiusman R. Studi Literasi: Pemahaman Konsep Anak Pada Pembelajaran Matematika. FIBONACCI J Pendidik Mat dan Mat. 2020;6(1):1.

2. Firmasari S, Herman T, Firdaus EF. Rigorous Mathematical Thinking: Conceptual Knowledge and Reasoning in the Case of Mathematical Proof. Kreano, J Mat Kreat. 2022;13(2):246–56.

3. Altıparmak, K., & Öziş T. An investigation upon mathematical proof and development of mathematical reasoning. Ege J Educ 6(1), 25–37. 2005. https://dergipark.org.tr/tr/pub/egeefd/issue/4918/67296

4. Halpern JY, Pucella R. A Logic for Reasoning about Evidence. J Artif Intell Res. 2006;25:1–34.

5. Hanna G. Proof, explanation, and exploration: An overview. Educ Stud Math 44, 5–23. 2000. https://doi.org/10.1023/A:1012737223465

6. Nance DA. The Best Evidence Principle. Case Western Reserve University: Faculty Publications; 1988. 463 p.

7. CadwalladerOlsker T. What Do We Mean by Mathematical Proof? J Humanist Math. 2011;1(1):33–60.

8. Helma, Murni D. PENGEMBANGAN LEMBARAN KERJA AKTIF MENGINTEGRASIKAN STATEMENT AND REASON UNTUK MENINGKATKAN KEMAMPUAN PEMECAHAN MASALAH. 2020;(184).

9. Kari Jabbour K. Multimedia Principle in Teaching Lessons. Acta Didact Napocensia. 2012;5(4):11–6.

10. Mayer R. e-Learning: Promise and Pitfalls. e-Learning and the Science of Instruction. 2016. 7–28 p.

11. Budiman H. Penggunaan Media Visual dalam Proses Pembelajaran, Al-Tadzkiyyah: J Pendidik Agama Islam. 2016;7(45):177.

12. Badruzaman, A., Nurdin, S., & Apriliya S. Pengaruh Penggunaan Media Visual Terhadap Hasil Belajar Siswa Pada Materi Peta. Pedadidaktika. J Ilm Pendidik Guru Sekol Dasar, 2(1), 118-128. 2015;

13. Weber K. Mathematics majors' evaluation of mathematical arguments and their conception of proof. Proc 12th Conf Res Undergrad Math Educ. 2009. http://sigmaa.maa.org/rume/crume2009/proceedings.html

14. Moore. College students' difficulties in learning to do mathematical proofs. Dr Diss Univ Georg Athens. 1990.

15. Mason. At.all. Thinking mathematically. Vol. 15, Early Years Educator. 2010. 24–44 p.

16. Faizah S, Nusantara T, Sudirman S, Rahardi R. Exploring students' thinking process in mathematical proof of abstract algebra based on Mason's framework. J Educ Gift Young Sci. 2020;8(2):871–84.

17. Öztürk T, Güven B. A study on pre-service mathematics teachers ' criteria of proof evaluation. 2022.

18. Martias. LD. Statistika Deskriptif Sebagai Kumpulan Informasi. Fihris J Ilmu Perpust dan Inf. 2021;16(1):40–59.

19. Weber.K. ). Student difficulty in constructing proofs: The need for strategic knowledge, Educational Studies in Mathematics, 48. 2001;101–119. https://link.springer.com/article/10.1023/A:1015535614355

20. de Villiers M. The role and function of proof in Mathematics. Vol. 24, Pythagoras. 1990. p. 17–23.

21. Muzangwa J, Ogbonnaya U. Visualization techniques for proofs: Implications for enhancing conceptualization and understanding in mathematical analysis. J Honai Math. 2024;7(2):347–62.

22. Kustandi C, Farhan M, Zianadezdha A, Fitri AK, L NA. Pemanfaatan Media Visual Dalam Tercapainya Tujuan Pembelajaran. Akademika. 2021;10(02):291–9.

23. Arnawa IM, Musdi E. Improving Students ' Mathematical Connections in Linear Programming through a GeoGebra-Supported Local Learning Design Based on Realistic Mathematics Education Mejorar las conexiones matemáticas de los estudiantes en programación lineal a través de un diseño de aprendizaje local respaldado por GeoGebra basado en una educación matemática realista. 2025;

24. Yerizon Y, Arnawa M, Arnellis A, Tasman F, Unp Y, Hevardani KA. Analysis of students' errors in drawing function graphs in three-dimensional space based on APOS theory. Salud, Cienc y Tecnol. 2025;5.

## FINANCING

The authors would like to thank Lembaga Penelitian dan Pengabdian Masyarakat Universitas Negeri Padang for funding this work with a contract number: 1675/UN35.15/LT/2024 fiscal year 2024.

## CONFLICT OF INTEREST

The author declares that there is no conflict of interest.

## AUTHORSHIP CONTRIBUTION

*Conceptualization:* Dewi Murni.
*Data curation:* Yerizon.
*Formal Analysis:* Elita Zusti Jamaan.
*Research*: Helma Helma.
*Methodology:* Made Arnawa.
*Project Management*: Yerizon.
*Resources*: Dewi Murni.
*Software*: Helma Helma.
*Supervision:* Eita Zusti Jamaan.
*Validation:* Made Arnawa.
*Appearance:* Mirna.
*Original draft*: Dewi Murni.
*Writing-proofreading and editing:* Mirna.