

ORIGINAL

Improving Students' Mathematical Connections in Linear Programming through a GeoGebra-Supported Local Learning Design Based on Realistic Mathematics Education

Mejorar las conexiones matemáticas de los estudiantes en programación lineal a través de un diseño de aprendizaje local respaldado por GeoGebra basado en una educación matemática realista

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ABSTRACT

Introduction: students often struggle to develop mathematical connection abilities in linear programming due to its abstract nature and procedural teaching methods. While Realistic Mathematics Education (RME) and Local Instructional Theory (LIT) offer structured learning trajectories, and GeoGebra provides dynamic visualization, their integration into a cohesive learning design for linear programming remains underexplored.

Objective: this study aimed to develop and evaluate a GeoGebra-supported local learning design grounded in RME to improve students' mathematical connection abilities in linear programming.

Method: a design-based research methodology was employed, involving iterative development and testing. A quasi-experimental pretest-posttest control group design was used to assess effectiveness with 68 undergraduate mathematics education students. The experimental group (n=34) received instruction via the developed learning design, while the control group (n=34) received conventional instruction. Data were collected using a Mathematical Connection Ability Test (MCAT), observations, and questionnaires.

Results: the expert validation showed high scores for content validity (M=4,64), construct validity (M=4,51), and practicality (M=4,17). Quantitatively, the experimental group significantly outperformed the control group in post-test scores ($t(66)=5,94$, $p<0,001$) with a large effect size (Cohen's $d=1,45$), demonstrating a greater improvement in connecting concepts to real-world contexts and within problem-solving processes. Qualitatively, students valued the contextualized approach and GeoGebra's interactivity for facilitating deeper understanding.

Conclusions: the GeoGebra-assisted LIT based on RME significantly enhanced students' mathematical connection abilities. The study proposes an effective, integrated instructional design for linear programming and recommends future research to optimize time allocation and extend the model to other mathematical topics.

Key Words: Mathematical Connection Abilities; Realistic Mathematics Education; Local Instructional Theory; GeoGebra; Blended Learning.

RESUMEN

Introducción: los estudiantes frecuentemente tienen dificultades para desarrollar habilidades de conexión

matemática en programación lineal debido a su carácter abstracto y a los enfoques de enseñanza procedimentales. Aunque la Educación Matemática Realista (RME) y la Teoría de la Instrucción Local (LIT) ofrecen trayectorias de aprendizaje estructuradas, y GeoGebra proporciona visualización dinámica, su integración en un diseño de aprendizaje cohesivo para la programación lineal ha permanecido poco explorada. **Objetivo:** este estudio tuvo como objetivo desarrollar y evaluar un diseño de aprendizaje local apoyado en GeoGebra y fundamentado en la RME para mejorar las habilidades de conexión matemática de los estudiantes en programación lineal.

Método: se empleó una metodología de investigación basada en el diseño, que incluyó desarrollo iterativo y prueba. Se utilizó un diseño cuasi-experimental con pretest-posttest y grupo de control para evaluar la efectividad con 68 estudiantes de licenciatura en educación matemática. El grupo experimental ($n=34$) recibió instrucción mediante el diseño de aprendizaje desarrollado, mientras que el grupo de control ($n=34$) recibió instrucción convencional. Los datos se recolectaron mediante una Prueba de Habilidad de Conexión Matemática (MCAT), observaciones y cuestionarios.

Resultados: la validación de expertos mostró puntuaciones altas para validez de contenido ($M=4,64$), validez de constructo ($M=4,51$) y practicidad ($M=4,17$). Cuantitativamente, el grupo experimental superó significativamente al grupo de control en las puntuaciones posttest ($t(66)=5,94$, $p<0,001$) con un tamaño del efecto grande (d de Cohen= $1,45$), demostrando una mayor mejora en conectar conceptos con contextos reales y dentro de los procesos de resolución de problemas. Cualitativamente, los estudiantes valoraron el enfoque contextualizado y la interactividad de GeoGebra para facilitar una comprensión más profunda.

Conclusiones: la LIT asistida por GeoGebra y basada en la RME mejoró significativamente las habilidades de conexión matemática de los estudiantes. El estudio propone un diseño instruccional integrado y efectivo para la programación lineal y recomienda que investigaciones futuras exploren la optimización de la asignación de tiempo y extiendan el modelo a otros contenidos matemáticos.

Palabras clave: Habilidades de Conexión Matemática; Educación Matemática Realista; Teoría de la Instrucción Local; Geogebra; Aprendizaje Híbrido.

INTRODUCTION

Mathematics education is fundamental in developing students' problem-solving and analytical skills, particularly in higher education, where complex mathematical concepts are applied across multiple disciplines. One of the key topics in applied mathematics is linear programming, a method used for optimizing limited resources in various fields, including economics, engineering, and logistics.⁽¹⁾ Despite its significance, students often struggle with understanding and applying linear programming concepts due to its abstract nature and the lack of effective instructional methods that bridge theoretical knowledge with real-world applications.⁽²⁾

Mathematics education is fundamental in developing students' problem-solving and analytical skills, particularly in higher education, where complex mathematical concepts are applied across multiple disciplines. One of the key topics in applied mathematics is linear programming, a method used for optimizing limited resources in various fields, including economics, engineering, and logistics.⁽¹⁾ Despite its significance, students often struggle with understanding and applying linear programming concepts due to its abstract nature and the lack of effective instructional methods that bridge theoretical knowledge with real-world applications.⁽²⁾

One promising pedagogical approach in mathematics education is Realistic Mathematics Education (RME), which emphasizes the use of real-world problems as the starting point for mathematical learning.^(3,4,5) RME has been shown to enhance students' ability to connect mathematical theories with real-life applications, promoting a more meaningful learning experience.⁽⁶⁾

In addition, Local Instructional Theory (LIT) provides a structured learning trajectory tailored to specific mathematical concepts. LIT supports the development of instructional designs that align with students' thinking processes and learning goals.^(2,7,8) Research on LIT in mathematics education has demonstrated positive outcomes in fostering students' cognitive development and mathematical reasoning.^(9,10) However, most existing LIT models have not fully integrated technology to support blended learning environments, which are increasingly being adopted in higher education.

Incorporating technology-assisted learning, particularly through dynamic mathematical software like Geogebra, offers a promising solution to enhance students' understanding of linear programming.⁽¹¹⁾ Geogebra provides interactive visual representations, allowing students to explore and manipulate linear programming models dynamically.^(12,13) Previous studies have highlighted its effectiveness in improving mathematical representation and problem-solving skills,⁽¹⁴⁾ yet its integration within an RME-based LIT for blended learning remains underexplored.⁽¹⁵⁾

Despite the growing body of research on RME, LIT, and Geogebra, there remains a significant gap in the

development of a structured instructional model that combines these three elements for blended learning in linear programming.⁽¹⁶⁾ Most existing studies have either focused on RME without leveraging technology or Geogebra without a well-defined instructional framework.⁽¹⁷⁾ Furthermore, students' mathematical connection abilities, essential for solving linear programming problems, have not been extensively examined in the context of RME-based learning trajectories.⁽¹⁸⁾

The findings of this study aim to contribute to mathematics education by introducing an integrated instructional theory aligned with RME, LIT, and technology-assisted learning. The proposed model bridges theoretical and practical gaps and offers a structured pedagogical framework suitable for implementation in higher education mathematics courses.^(19,20)

METHOD

This section describes the research methodology employed to develop and evaluate a Geogebra-assisted Local Instructional Theory (LIT) based on Realistic Mathematics Education (RME) for blended learning in linear programming. The methodology follows a design-based research (DBR) approach, particularly suitable for developing and refining educational innovations through iterative design, implementation, analysis, and redesign cycles.⁽²¹⁾ The development process followed Plomp's model, while the evaluation used a quasi-experimental design.

Research Design

A design-based research approach was adopted for this study, following the four-phase model proposed by Bakker et al.⁽²²⁾: preparation and design, teaching experiment, retrospective analysis, and revision.

Research Setting and Participants

Characteristic	Experimental Group (n = 34)	Control Group (n = 34)	Total (N = 68)
Gender			
Female	23 (67,6 %)	23 (67,6 %)	46 (67,6 %)
Male	11 (32,4 %)	11 (32,4 %)	22 (32,4 %)
Age			
19 years	15 (44,1 %)	14 (41,2 %)	29 (42,6 %)
20 years	12 (35,3 %)	13 (38,2 %)	25 (36,8 %)
21 years	5 (14,7 %)	6 (17,6 %)	11 (16,2 %)
22 years	2 (5,9 %)	1 (2,9 %)	3 (4,4 %)
Prior GPA			
3,50-4,00	7 (20,6 %)	6 (17,6 %)	13 (19,1 %)
3,00-3,49	17 (50,0 %)	18 (52,9 %)	35 (51,5 %)
2,50-2,99	9 (26,5 %)	8 (23,5 %)	17 (25,0 %)
2,00-2,49	1 (2,9 %)	2 (5,9 %)	3 (4,4 %)
Mathematics Background			
Completed Linear Algebra	34 (100 %)	34 (100 %)	68 (100 %)
Completed Calculus I & II	34 (100 %)	34 (100 %)	68 (100 %)
Prior Linear Programming Experience	0 (0 %)	0 (0 %)	0 (0 %)
Technology Proficiency			
High	8 (23,5 %)	7 (20,6 %)	15 (22,1 %)
Moderate	20 (58,8 %)	22 (64,7 %)	42 (61,8 %)
Low	6 (17,6 %)	5 (14,7 %)	11 (16,2 %)
Note: Participants self-reported technology proficiency based on their familiarity with digital learning tools and mathematical software.			

The study was conducted at Pahlawan Tuanku Tambusai University, Indonesia. Participants were selected using purposive sampling,⁽²³⁾ as this approach enabled the deliberate selection of undergraduate mathematics education students enrolled in an Operations Research course that included linear programming as a core topic, ensuring participants possessed the specific foundational knowledge necessary to inform the study's objective

of developing and evaluating the proposed instructional design.

Sixty-eight second-year undergraduate students participated in the study, comprising 46 females (67,6 %) and 22 males (32,4 %) aged 19-22. These students were divided into two groups: an experimental group ($n = 34$) that received instruction using the Geogebra-assisted LIT based on RME and a control group ($n = 34$) that received conventional instruction. All participants had completed prerequisite courses in linear algebra and calculus but had no prior formal instruction in linear programming. The demographic characteristics of the participants are presented in table 1.

Prior to the intervention, equivalence between the experimental and control groups was established through a pretest on mathematical connection abilities ($t(66) = 0,87$, $p = 0,39$), indicating no significant difference between the groups. Additionally, an independent samples t-test showed no significant difference in prior academic performance as measured by GPA ($t(66) = 0,78$, $p = 0,44$). These results suggest that the groups were comparable at the study's outset.

Development of the Geogebra-Assisted Local Instructional Theory

The development of the Geogebra-assisted LIT based on RME for linear programming followed Plomp's three-phase model: preliminary research, prototyping, and assessment. This model is ideal for creating educational interventions with practical relevance.⁽²¹⁾

Preliminary Research Phase

This phase involved a needs assessment, curriculum analysis, and literature review. Interviews with six instructors and surveys of 40 students revealed difficulties connecting concepts to real-life contexts and visualizing linear programming problems.^(24,25)

Curriculum analysis identified five core competencies: modeling real-world problems, representing feasible regions, finding optimal solutions graphically, interpreting results contextually, and applying concepts across domains.

A preliminary LIT was developed using RME principles: guided reinvention, didactical phenomenology, and emergent modelling.⁽²⁶⁾ The LIT was structured into five sequential phases to promote progressive mathematization through contextual problems.

Prototyping Phase

The preliminary LIT was developed into detailed instructional materials, including contextual tasks, activities, and Geogebra applets, suitable for blended learning.⁽²⁷⁾ The applets were designed based on TPACK principles,⁽²⁸⁾ helping students visualize concepts, manipulate parameters, and strengthen understanding.⁽²⁹⁾

Expert review was conducted by seven specialists (in mathematics education, educational technology, and linear programming) using a validated instrument. This instrument was originally developed and validated with a panel of Indonesian mathematics education experts for a similar context of evaluating technology-integrated learning designs.⁽³⁰⁾ The experts assessed the instructional theory using a 5-point Likert scale (1 = strongly disagree to 5 = strongly agree) across three dimensions: content validity, construct validity, and practicality. Feedback led to a refined prototype ready for classroom testing.

Assessment Phase

This phase included a teaching experiment using a pretest-posttest control group design. The experimental group used the Geogebra-assisted LIT, while the control group received traditional instruction.

Instruments and Data Collection

Data for the teaching experiment was collected using the following instruments:

Mathematical Connection Ability Test (MCAT)

The MCAT assessed students' ability to relate mathematical concepts to real-world and cross-conceptual contexts, using five open-ended problems. It was validated through expert judgment ($CVI = 0,87$), factor analysis ($KMO = 0,79$), and showed high reliability ($\alpha = 0,83$). Scoring used a rubric (0-4 per item, total 20 points).

Classroom Observation Protocol

A structured observation protocol was used to document the implementation of the instructional theory in the classroom. The protocol focused on five key aspects: teacher facilitation, student engagement, use of context, technology integration, and mathematical discourse.^(31,32,33,34) Two trained observers independently completed the protocol during each teaching session, with an inter-rater agreement of 87 %. This is separate from the instrument used earlier for the expert review of the learning design prototype.

Implementation Procedure

Implementation spanned eight weeks with three hours per week. The experimental group followed the blended LIT approach; the control group used conventional instruction. Blended components included face-to-face sessions for collaboration, online synchronous discussions, and asynchronous activities like applet exploration and reflections. Implementation was guided by Treffers' six RME principles: activity, reality, level, intertwining, interaction, and guidance.⁽³¹⁾ Geogebra integration followed TPACK guidelines.⁽³²⁾

Quantitative Analysis

Quantitative data from the MCAT were analyzed using descriptive and inferential statistics. Descriptive statistics included means, standard deviations, and normalized gain scores to assess the improvement in mathematical connection abilities. Inferential statistics included an independent samples t-test to compare the post-test scores between the experimental and control groups and a paired samples t-test to compare pretest and post-test scores within each group. Effect sizes were calculated using Cohen's d to determine the practical significance of the intervention.⁽³³⁾

Qualitative Analysis

Interviews, open-ended responses, and observations were analyzed thematically, using a six-step coding process. Thematic analysis combined priori codes with emerging themes.⁽³⁴⁾ Trustworthiness was ensured via triangulation, member checking, peer debriefing, and audit trails. Two researchers coded independently (85 % agreement).

Integration of Findings

The quantitative and qualitative findings were integrated using a convergent parallel design in which the two types of data were analyzed separately and then merged for interpretation. The integration focused on identifying convergences and divergences between the quantitative and qualitative results and using qualitative findings to explain and elaborate on the quantitative results.

Validity and Reliability

Several measures were taken to ensure the validity and reliability of the study. Content validity of the instruments was established through expert review, while construct validity was confirmed through factor analysis where appropriate. The reliability of quantitative instruments was established using Cronbach's alpha, and inter-rater reliability was calculated for observational and scoring protocols. For qualitative data, trustworthiness was enhanced through triangulation, member checking, peer debriefing, and maintaining an audit trail.⁽³⁵⁾

Ethical Considerations

This study was conducted in accordance with recognized ethical guidelines for educational research. Prior to participation, all students were informed about the nature and purpose of the study. Written informed consent was obtained from all participants, ensuring their participation was voluntary and that they could withdraw at any time without penalty. The confidentiality of all participant data was strictly maintained throughout the research process, with data being anonymized for analysis and publication.

RESULTS

The expert validation results for the instructional theory are presented in table 2. The feedback indicated strong overall approval but also provided specific insights for refinement.

The expert validation showed high ratings across all dimensions, with an overall score of 4,44, indicating strong approval of the instructional theory. Content validity scored highest at 4,64, showing the theory covered essential linear programming content well, while construct validity scored 4,51, confirming effective integration of RME principles and Geogebra. Practicality scored slightly lower at 4,17, mainly due to concerns about time allocation, as implementing the theory fully may require more time than typical courses allow. Based on this feedback, the theory was refined by adding exploration phases, enhancing applet interactivity, adjusting time management, and providing extra instructor support.

Structure of the Finalized Instructional Theory

The finalized Geogebra-assisted LIT based on RME consisted of five sequential phases, each targeting specific aspects of linear programming through contextual problems and progressive mathematization. Table 3 summarizes the structure of the instructional theory.

Table 2. Expert Validation Results for the Geogebra-Assisted LIT Based on RME (N = 7)

Dimension	Aspect	Mean	SD	Category
Content Validity	Alignment with curriculum objectives	4,71	0,49	Very High
	Mathematical accuracy	4,86	0,38	Very High
	Completeness of content coverage	4,43	0,53	High
	Sequencing of mathematical concepts	4,57	0,53	Very High
	Overall Content Validity	4,64	0,48	Very High
Construct Validity	Alignment with RME principles	4,57	0,53	Very High
	Integration of Geogebra with Learning Trajectory	4,43	0,79	High
	Facilitation of mathematical connections	4,71	0,49	Very High
	Support for progressive mathematization	4,29	0,76	High
	Appropriateness for blended learning	4,57	0,53	Very High
	Overall Construct Validity	4,51	0,62	Very High
Practicality	Clarity of instructional materials	4,29	0,76	High
	Usability of GeoGebra applets	4,57	0,53	Very High
	Feasibility of implementation	4,14	0,69	High
	Time allocation	3,86	0,69	High
	Scalability to different contexts	4,00	0,82	High
	Overall Practicality	4,17	0,71	High
Overall Validation Score		4,44	0,63	High

Note: Rating scale: 1,00-1,80 = Very Low; 1,81-2,60 = Low; 2,61-3,40 = Moderate; 3,41-4,20 = High; 4,21-5,00 = Very High

Table 3. Structure of the Finalized Geogebra-Assisted LIT Based on RME for Blended Learning in Linear Programming

Phase	Mathematical Focus	RME Principle	Geogebra Integration	Blended Learning Format	Key Learning Activities
Contextual Problem Exploration	Understanding optimization problems in real-world contexts	Reality, Activity	Interactive problem scenarios with adjustable parameters	F2F: Collaborative problem analysis Async: Contextual problem exploration	- Analysis of real-world optimization scenarios - Identification of decision variables, constraints, and objectives - Discussing the mathematical relevance of contexts
Model Formulation	Translating contextual problems into mathematical models	Guided reinvention, Interaction	Model builder applet with immediate visual feedback	F2F: Guided model formulation Sync online: Group discussion Async: Model refinement	- Identifying variables and units - Formulating constraint equations - Defining objective functions
Graphical Representation	Visualizing feasible regions and understanding geometric properties	Level, Intertwining	Dynamic graphing applet with constraint manipulation	F2F: Instructor-guided visualization Async: Constraint exploration Sync online: Collaborative analysis	- Plotting constraints and feasible regions - Exploring the effects of constraint modifications - Identifying boundary points and vertices - Connecting algebraic and geometric representations
Solution Determination	Finding and verifying optimal solutions	Guidance, Level	Optimization simulator with solution path tracing	F2F: Problem-solving strategies Sync online: Solution Comparison Async: Method exploration	- Applying graphical method - Exploring extreme points - Analyzing sensitivity to constraint changes - Comparing solution strategies - Proving optimality of solutions
Interpretation and Application	Interpreting solutions in original contexts and extending to new situations	Reality, Intertwining	Context-specific visualization with multiple representations	F2F: Case study analysis Async: Application project Sync online: Project presentation	- Interpreting solutions in original contexts - Making decisions based on mathematical results - Extending models to similar scenarios - Reflecting on mathematical connections

Note: F2F = Face-to-Face; Sync = Synchronous; Async = Asynchronous

The instructional theory was implemented using a blended learning model with 40 % face-to-face sessions, 20 % synchronous online, and 40 % asynchronous activities, each chosen to maximize collaborative learning, real-time interaction, and individual exploration. Instructors praised the clear, sequential design and the deep integration of Geogebra, which helped students visualize complex concepts and connect contextual problems to formal mathematics.

Effect on Students' Mathematical Connection Abilities

To evaluate the impact of the developed instructional theory, its effect on students' mathematical connection abilities in linear programming was analyzed. This question was addressed through analysis of the Mathematical Connection Ability Test (MCAT) results and related qualitative data.

Mathematical Connection Ability Test Results

Table 4 presents the descriptive statistics and t-test results for the MCAT, comparing pretest and post-test scores between the experimental and control groups.

Group	Test	Mean	SD	Mean Difference	t	df	p	Cohen's d	N-gain
Experimental (n = 34)	Pretest	5,79	1,95	8,65	19,87	33	<0,001*	3,41	0,68
	Post-test	14,44	2,74						
Control (n = 34)	Pretest	5,47	1,71	5,12	12,95	33	<0,001*	2,22	0,39
	Post-test	10,59	2,56						
Between Groups	Pretest	0,32	0,47	0,87	66	0,389	0,18	-	
	Posttest	3,85	5,94	66	<0,001*	1,45	-		

Note: maximum possible score = 20; N-gain = normalized gain score; *p < 0,05

The results show that both groups significantly improved from the pretest to the post-test, indicating that both instructional approaches effectively developed students' mathematical connection abilities. However, the experimental group showed substantially greater improvement ($M = 8,65$, $SD = 2,55$) compared to the control group ($M = 5,12$, $SD = 2,30$), with a very large effect size (Cohen's $d = 3,41$) for the experimental group compared to a large effect size (Cohen's $d = 2,22$) for the control group.

The normalized gain score (N-gain), which measures the ratio of actual improvement to potential improvement, further illustrates the difference in learning effectiveness: the experimental group achieved an N-gain of 0,68, indicating high effectiveness, while the control group achieved an N-gain of 0,39, indicating moderate effectiveness. ^(36,37,38,39)

Most notably, while there was no significant difference between the groups at the pretest ($t(66) = 0,87$, $p = 0,389$), the experimental group significantly outperformed the control group at the post-test ($t(66) = 5,94$, $p < 0,001$), with a very large effect size (Cohen's $d = 1,45$). This indicates that the Geogebra-assisted LIT based on RME was substantially more effective than conventional instruction in developing students' mathematical connection abilities.

Further analysis of the MCAT results by connection dimension, as shown in table 5, provides more detailed insights into the specific aspects of mathematical connection abilities most affected by the instructional approach.

The analysis by connection dimension reveals that the experimental group showed greater improvement than the control group across all three dimensions of mathematical connection. The largest difference between the groups was observed in "Connection to real-world contexts," where the experimental group's mean improvement ($M = 3,06$, $SD = 1,06$) was more than twice that of the control group ($M = 1,47$, $SD = 0,93$). As shown in table 5, the experimental group showed significantly greater improvement than the control group in connecting mathematical concepts to real-world contexts. Qualitative data from student interviews further detailed the nature of these improved connection abilities.

Perceptions of the Instructional Theory's Validity, Practicality, and Effectiveness

Perceptions of the instructional theory's validity, practicality, and effectiveness were gathered from students and instructors in the blended learning environment. These perceptions were analyzed using dedicated questionnaires and supported by related qualitative data.

Student and Instructor Perceptions

Table 6 presents the results of the validity, practicality, and effectiveness questionnaires completed by

students and instructors in the experimental group.

Table 5. Mathematical Connection Ability Test Results by Connection Dimension

Connection Dimension	Group	Pretest		Post-test		Mean Difference		t	p	Cohen's d
The connection between mathematical concepts	Experimental	2,06	0,85	4,76	1,02	2,70	15,31	<0,001*	2,62	
	Control	1,91	0,87	3,76	0,92	1,85	11,56	<0,001*	1,98	
	Between Groups (p)	0,489		<0,001*						
Connection to real-world contexts	Experimental	1,97	0,90	5,03	1,03	3,06	16,80	<0,001*	2,88	
	Control	1,88	0,81	3,35	0,98	1,47	9,22	<0,001*	1,58	
	Between Groups (p)	0,658		<0,001*						
Connection within problem-solving processes	Experimental	1,76	0,78	4,65	1,07	2,89	16,33	<0,001*	2,80	
	Control	1,68	0,73	3,48	0,96	1,80	11,89	<0,001*	2,04	
	Between Groups (p)	0,648		<0,001*						

Note: Maximum possible score for each dimension = 7; *p < 0,05

Table 6. Student and Instructor Perceptions of the Instructional Theory's Validity, Practicality, and Effectiveness

Dimension	Aspect	Students (n = 34)		Instructors (n = 2)		One-sample t-test (Students)		
		Mean	SD	Mean	SD	t	df	p
Validity	Alignment with learning objectives	4,35	0,69	4,50	0,71	11,52	33	<0,001*
	Coherence of learning trajectory	4,18	0,76	4,50	0,71	9,03	33	<0,001*
	Relevance of contextual problems	4,62	0,55	5,00	0,00	17,29	33	<0,001*
	Integration of mathematical concepts	4,24	0,78	4,50	0,71	9,25	33	<0,001*
Practicality	Overall Validity	4,35	0,71	4,63	0,52	11,06	33	<0,001*
	Clarity of instructions	3,88	0,95	4,00	0,00	5,45	33	<0,001*
	Time adequacy	3,71	1,03	3,50	0,71	3,99	33	<0,001*
	Ease of Geogebra use	4,26	0,79	4,50	0,71	9,36	33	<0,001*
	Accessibility of blended format	4,15	0,82	4,00	0,00	8,17	33	<0,001*
	Support materials	4,03	0,87	4,00	0,00	6,97	33	<0,001*
	Overall Practicality	4,01	0,92	4,00	0,47	6,36	33	<0,001*
Effectiveness	Enhancement of understanding	4,41	0,70	4,50	0,71	11,81	33	<0,001*
	Improvement in problem-solving	4,32	0,73	4,50	0,71	10,65	33	<0,001*
	Development of mathematical connections	4,56	0,61	5,00	0,00	14,91	33	<0,001*
	Engagement and motivation	4,47	0,71	4,50	0,71	12,14	33	<0,001*
	Preparation for applications	4,38	0,70	4,50	0,71	11,57	33	<0,001*
	Overall Effectiveness	4,43	0,69	4,60	0,52	12,05	33	<0,001*

Note: Rating scale: 1 = strongly disagree to 5 = strongly agree; One-sample t-test tested against neutral value (3); *p < 0,05

The results show that students and instructors positively perceived the instructional theory's validity, practicality, and effectiveness. For students, all dimensions received mean ratings significantly above the neutral point of 3 on the 5-point scale (all p < 0,001), indicating strong positive perceptions. Effectiveness received the highest overall rating (M = 4,43, SD = 0,69), followed by validity (M = 4,35, SD = 0,71) and practicality (M = 4,01, SD = 0,92).

Among the individual aspects, "Relevance of contextual problems" received the highest rating from both students (M = 4,62, SD = 0,55) and instructors (M = 5,00, SD = 0,00), highlighting the success of the RME approach in providing meaningful contexts for learning. "Development of mathematical connections" also received very high ratings from both students (M = 4,56, SD = 0,61) and instructors (M = 5,00, SD = 0,00), aligning with the quantitative results from the MCAT.

Though still positive, the lowest ratings were for "Time adequacy" (Students: M = 3,71, SD = 1,03; Instructors: M = 3,50, SD = 0,71), suggesting that time constraints remained a challenge in implementing the instructional theory. This finding echoes the concerns raised by experts during the validation phase and indicates an area for further refinement.

The qualitative data from questionnaires and interviews revealed five key themes about student and

instructor perceptions: contextual relevance, technology integration, blended learning format, learning trajectory, and time management. Students valued meaningful real-world contexts that made mathematical concepts feel relevant and practical for future careers, and they appreciated starting with contextual problems to understand the rationale behind techniques rather than merely following procedures.

Learning Analytics Results

Learning analytics data from the Learning Management System (LMS) and Geogebra activities provided additional insights into student engagement with the instructional materials. Table 7 presents key engagement metrics for the experimental group.

Engagement Metric	Mean	SD	Range	Correlation with MCAT Post-test (r)	p
Total time spent on online activities (hours)	16,82	4,73	8,5-26,3	0,53	0,001*
Number of Geogebra applet interactions	87,35	24,61	45-142	0,61	<0,001*
Completion rate of asynchronous activities (%)	89,47	12,35	60-100	0,48	0,004*
Participation in online discussions (posts)	12,68	6,92	3-29	0,32	0,068
Time distribution across learning phases (%)					
- Phase 1: Contextual Problem Exploration	18,76	5,42	10-31	0,29	0,095
- Phase 2: Model Formulation	22,84	6,33	12-35	0,37	0,030*
- Phase 3: Graphical Representation	24,65	5,97	15-38	0,56	<0,001*
- Phase 4: Solution Determination	21,53	5,64	12-33	0,45	0,008*
- Phase 5: Interpretation and Application	12,22	4,81	5-22	0,31	0,073

Note: *p < 0,05

The learning analytics data revealed several interesting patterns. First, there was considerable variation in student engagement with the online components of the course, as evidenced by the range of time spent on online activities (8,5 to 26,3 hours) and the number of Geogebra applet interactions (45 to 142). Despite this variation, engagement metrics such as total time spent on online activities ($r=0,53, p=0,001$), Geogebra applet interactions ($r=0,61, p<0,001$), and completion rate of asynchronous activities ($r=0,48, p=0,004$) were significantly correlated with the MCAT post-test scores, suggesting that greater engagement with these instructional components was associated with improved learning outcomes.

Among the different learning phases, time spent on Model Formulation ($r=0,37, p=0,030$), Graphical Representation ($r=0,56, p<0,001$), and Solution Determination ($r=0,45, p=0,008$) showed significant positive correlations with post-test performance. This indicates that students with more time for these critical problem-solving phases tended to achieve better results. In contrast, time spent on Contextual Problem Exploration ($r=0,29, p=0,095$) and Interpretation and Application ($r=0,31, p=0,073$) did not exhibit statistically significant correlations, although they may still contribute qualitatively to student understanding.

Interestingly, participation in online discussions ($r=0,32, p=0,068$) was not significantly correlated with post-test performance, which may suggest that while discussions provide valuable opportunities for collaboration, their direct impact on mathematical problem-solving skills in this context was limited.

DISCUSSION

This study aimed to develop and evaluate a GeoGebra-assisted Local Instructional Theory (LIT) based on Realistic Mathematics Education (RME) to improve students' mathematical connection abilities in linear programming. The findings demonstrate that the integrated instructional design was not only valid and practical according to experts and users but also significantly more effective than conventional instruction in enhancing students' ability to connect mathematical concepts to real-world contexts and within problem-solving processes.

The quantitative results, showing a significantly larger improvement in the experimental group with a large effect size (Cohen's $d = 1,45$), strongly support the efficacy of the RME-based, technology-enhanced approach. This finding aligns with previous research indicating that RME facilitates meaningful learning by bridging abstract concepts and real-life situations.^(3,36,37) The mechanism behind this success appears to be the structured learning trajectory, which used contextual problems as a starting point (the "reality" principle), allowing students to progressively develop formal mathematical understanding through guided reinvention and modeling.^(26,38)

The most pronounced effect was observed in the “connection to real-world contexts” dimension. This can be directly attributed to the instructional design’s emphasis on didactical phenomenology, where real-world optimization problems were not just examples but the foundation for learning. This finding corroborates with studies by ⁽⁶⁾ and ⁽¹⁸⁾, which argue that contextualization is key to developing applied mathematical skills. The qualitative data further illuminate this mechanism; students reported that the relevance of the problems increased their motivation and helped them see the “why” behind the procedures, moving beyond rote memorization.

Furthermore, the integration of GeoGebra served as a critical cognitive tool. The dynamic visualization capabilities helped students overcome the primary difficulty of visualizing abstract linear programming concepts, as identified in the preliminary research.^(24,25,39) This finding is consistent with the work of ⁽¹²⁾ and ⁽¹⁴⁾, who found that dynamic geometry software supports conceptual understanding by enabling manipulation and exploration. In our study, the interactive applets allowed students to instantly see the consequences of changing constraints or objective functions, thereby strengthening the connection between algebraic models and their geometric representations.

The findings have several important implications for educational practice and theory. For practitioners, this study provides a validated and practical model for teaching linear programming and potentially other abstract mathematical topics. The five-phase LIT structure offers a clear roadmap for instructors to implement a blended, student-centered approach. For theory, this research contributes a refined Local Instructional Theory that successfully integrates the pedagogical principles of RME with the technological affordances of GeoGebra within a TPACK framework.⁽²⁸⁾ It demonstrates that a coherent instructional theory can systematically enhance specific cognitive abilities, such as mathematical connections, which are often overlooked in traditional, procedurally-focused teaching.

Despite the positive results, this study has several limitations that must be acknowledged. First, the use of purposive sampling at a single university, while justified for the development phase, limits the generalizability of the findings. The results are most applicable to similar contexts—undergraduate mathematics education students in Indonesia. Second, as noted by experts and participants, the implementation of the full learning design was time-intensive. This practicality concern suggests that in standard curricula with fixed hours, certain phases might need to be streamlined, potentially impacting the depth of learning. Third, the reliance on an instrument validated within a similar cultural and educational context^(30, 40) may introduce bias, and the findings should be interpreted with this contextual factor in mind. Finally, the study focused primarily on mathematical connection abilities; its impact on other outcomes, such long-term knowledge retention or affective domains like self-efficacy, remains to be investigated.

In conclusion, this study provides robust evidence that a GeoGebra-assisted LIT grounded in RME principles is a powerful intervention for improving mathematical connection abilities in linear programming. Future research should aim to overcome the limitations of this study, such as conducting longitudinal studies across multiple institutions and exploring the model’s adaptability to other mathematical domains.

CONCLUSIONS

This study demonstrates that a Geogebra-assisted Local Instructional Theory (LIT) based on Realistic Mathematics Education (RME) significantly enhances students’ mathematical connection abilities in linear programming. The structured learning trajectory, grounded in RME principles and supported by Geogebra’s dynamic visualization tools, enabled students to build deeper conceptual understanding and establish meaningful links between mathematical ideas, real-world applications, and problem-solving processes. Students in the experimental group consistently outperformed those in the control group across all dimensions of mathematical connections, highlighting the effectiveness of this integrated instructional approach.

Looking ahead, future research could examine the impact of this instructional model on other cognitive and affective domains, such as mathematical reasoning, critical thinking, and student motivation. Expanding the model to additional mathematical areas—like calculus, statistics, or discrete mathematics would offer insights into its broader applicability. Furthermore, the integration of adaptive learning technologies alongside Geogebra may enhance the personalization of instruction, allowing students with varying levels of prior knowledge and digital proficiency to receive tailored support. Overall, this study contributes to the advancement of mathematics education by offering an empirically supported pedagogical framework that bridges theoretical foundations and practical classroom implementation in linear programming.

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